## WEEK 1

1. Design ✓ predator-prey
2. Vensim ✓
3. R -> ggplot2 ✓
   -> Rmarkdown ✓

$$\begin{bmatrix} x \\ y \end{bmatrix}' = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$X' = AX + F \to 0$$

↑

Pascal Matrix → exp( )

Jim Duggan
Jair Andrade

tidyverse

→ readsdr

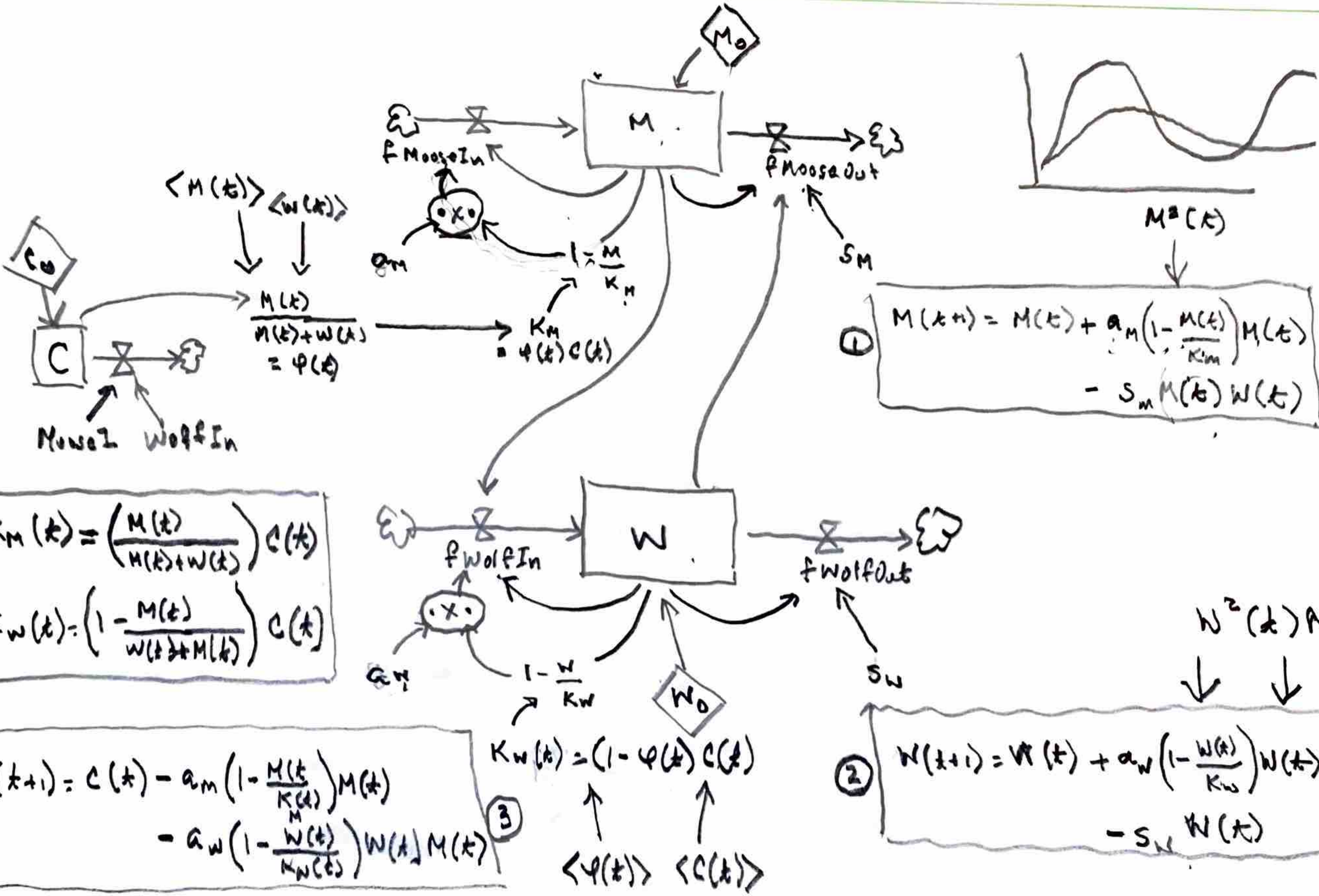
## WEEK 2

1. Design P-P WITH Limits to growth ⊕ ENDOGENIZE THE LIMITATION
2. Vensim
3. R model → deSolve
4. INFERENCE
   R → GENERATIVE MODEL
   → GRID APPROX

## WEEK 3 ?

1. TWEAK P-P
[2. VENSIM]
   ↓
3. R → ODE
   → GEN
   → INFER
4. STAN

# ENDOGENIZED LIMITS TO INTERACTIVE GROWTH



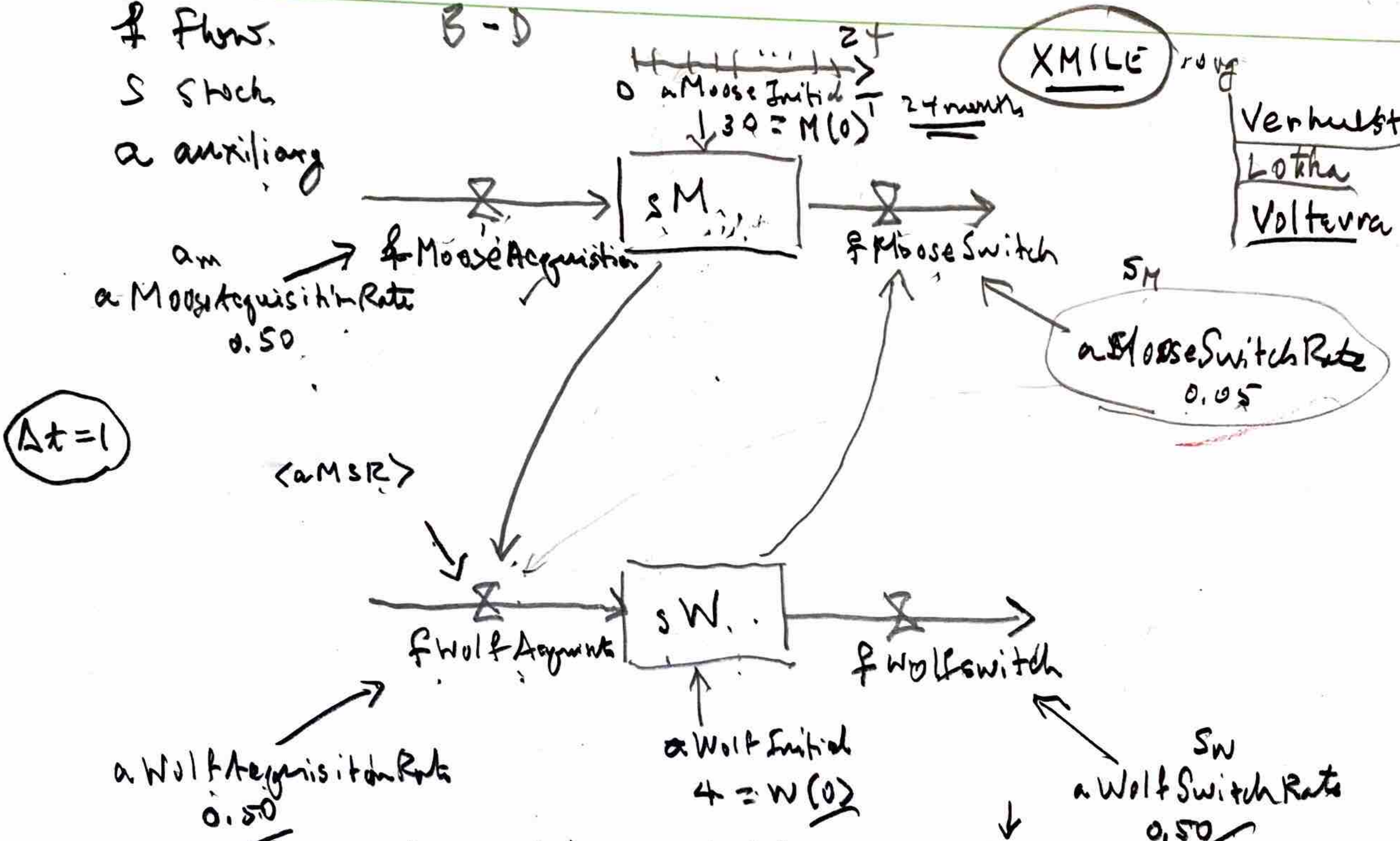

① $$M(t+1) = M(t) + a_M\left(1 - \frac{M(t)}{K_M}\right)M(t) - S_M M(t) W(t)$$

② $$W(t+1) = W(t) + a_W\left(1 - \frac{W(t)}{K_W}\right)W(t)M(t) - S_W W(t)$$

③ $$C(t+1) = C(t) - a_M\left(1 - \frac{M(t)}{K_M(t)}\right)M(t) - a_W\left(1 - \frac{W(t)}{K_W(t)}\right)W(t)M(t)$$

④ $$K_M(t) = \left(\frac{M(t)}{M(t)+W(t)}\right)C(t)$$

$$K_W(t) = \left(1 - \frac{M(t)}{W(t)+M(t)}\right)C(t)$$

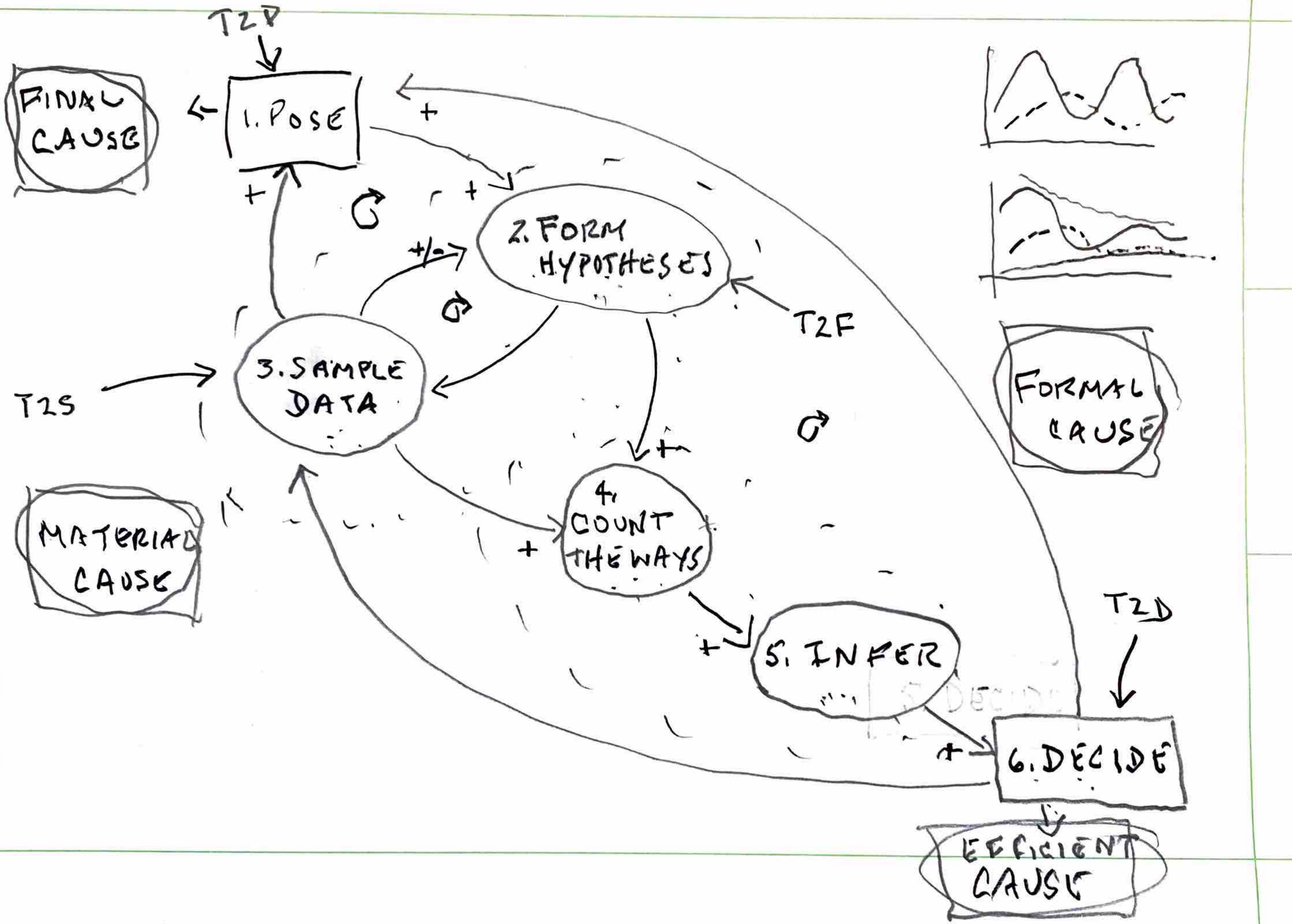



$$M(t+1) = M(t) + a_m M(t) - s_m M(t) W(t)$$

$$W(t+1) = W(t) + s_m a_w M(t) W(t) - s_w W(t)$$

Ex by hand

t = 1 to 5



T2P
FINAL CAUSE
1. POSE
2. FORM HYPOTHESES
T2F
3. SAMPLE DATA
T2S
4. COUNT THE WAYS
5. INFER
T2D
6. DECIDE
FORMAL CAUSE
MATERIAL CAUSE
EFFICIENT CAUSE
+
+/-

Ⓞ Pose Q ① Form h ② Sample d ③ Count w ④ Infer Pr

SYNTH BINARY

OBJECTIVE
HYPOTHETICAL
DEDUCTIVE

② 2 Factor collection of d

RELEVANT?

ENOUGH STUDENTS?

AVAILABLE INSTRUCTOR?

| | T | F |
|---|---|---|
| T | OPEN | CLOSE |
| F | CLOSE | CLOSE |

BOTH-AND

I ∧ S

CONJUNCTION

COLLECT (SAMPLE)

| 4/30 | 7/15 | 8/25 |
|---|---|---|
| C | O | C |
| Ø | 1 | Ø |

② d

Ø , 1 , Ø

① h

| | |
|---|---|
| 1 | Ø |
| Ø | 4 |
| 1 | 3 |
| 2 | 2 |
| 3 | 1 |
| 4 | Ø |

③ w

| | | | | |
|---|---|---|---|---|
| 4 | × | Ø | × | 4 |
| 3 | × | 1 | × | 3 |
| 2 | × | 2 | × | 2 |
| 1 | × | 3 | × | 1 |
| 0 | × | 4 | × | 0 |

④ Pr

| | |
|---|---|
| Ø | 0% |
| 9 | 45% |
| 8 | 40% |
| 3 | 15% |
| Ø | 0% |
| 20 | |

EITHER-OR

$h_0 \vee h_1 \vee h_2 \vee h_3 \vee h_4$

DISJUNCTION

FREQUENCY = COUNTS

④ Infer

CLOSE!

OBJ: MOST PLAUSIBLE

DEC: OPEN OR CLOSE

CON: C-O-C DATA & h GRID

PLAUSIBILITY = Pr( h = 25% | d = 0 1 0 )

RELATIVE FREQUENCY NORMALIZATION

SUFFICIENT?

MAKE SENSE?

① h GRID

COMPLETE?

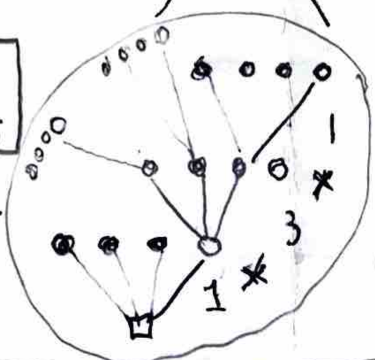
| OPEN | CLOSE | OPEN P |
|---|---|---|
| Ø | 4 | 0% |
| 1 | 3 | 25% |
| 2 | 2 | 50% |
| 3 | 1 | 75% |
| 4 | Ø | 0% |

5 vertex (nodes)

4 edges (intervals)

③ COUNTS

(LOGICAL CONJUNCTIONS OF h & d)




MULTIPLICATION OF BOTH AND AND

1 × 3 × 1

STORY

ON 4/30 I available but S not enough

ON 7/15 Both I & S!

ON 8/25 I quits and S fake

Ⓞ Should we open or close?

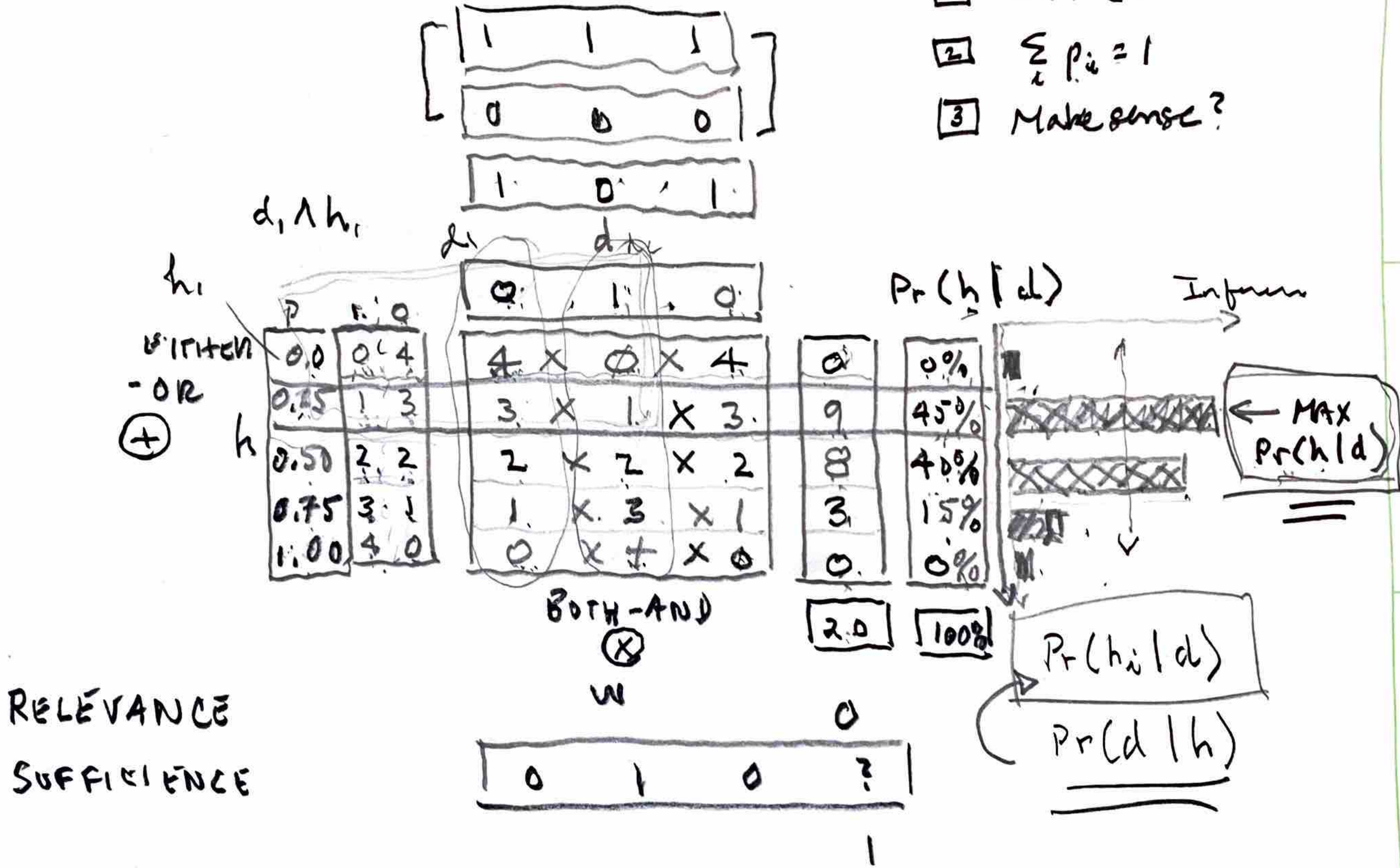
227/305

Fri (08-31)

DECISION PROCESS

①

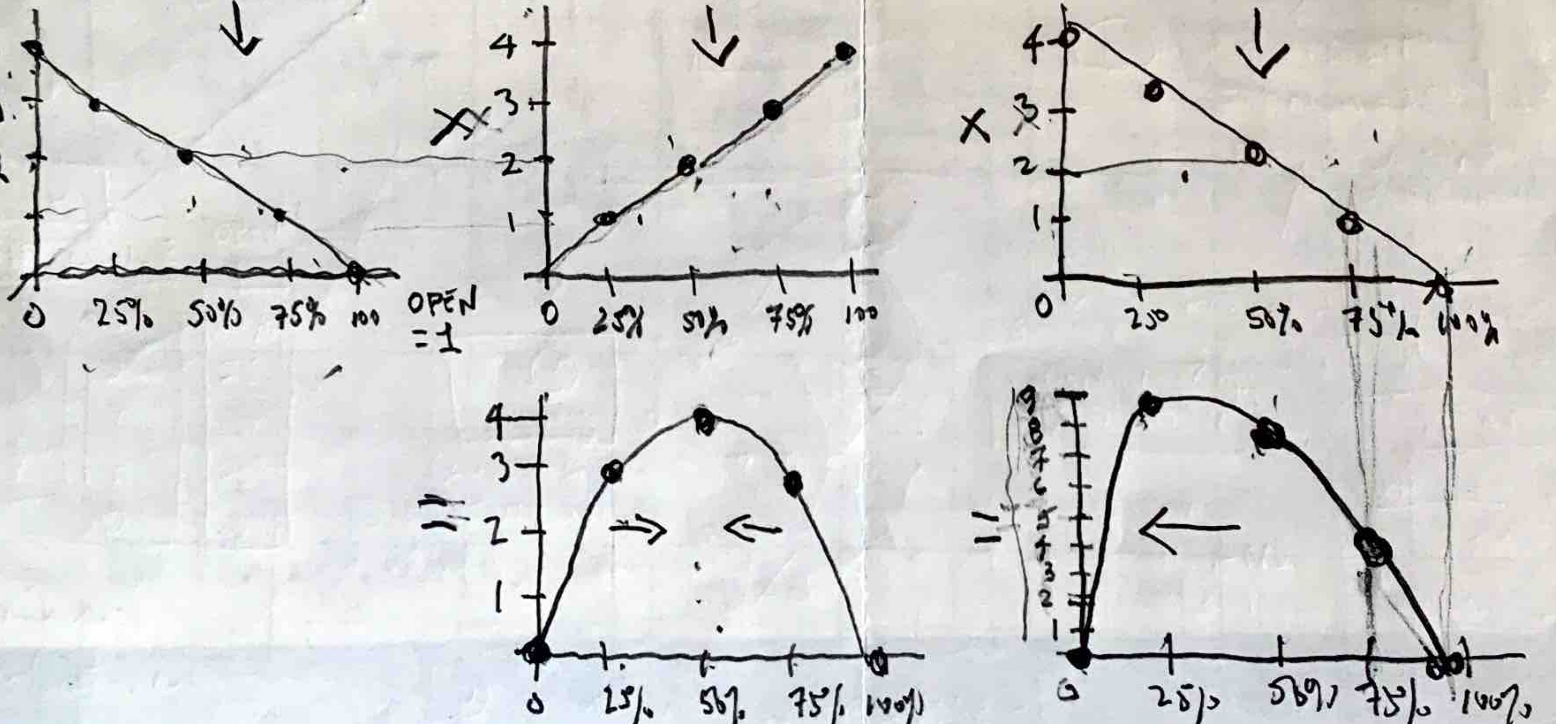
1. $0 \le p \le 1$
2. $\sum_i p_i = 1$
3. Make sense?



| p | 1 | 0 | 4/30 Ø | | × 7/15 1 | = | |
|---|---|---|---|---|---|---|---|
| 0% | 0 | 4 | 4 | 40% | 0 | 0 | 0% |
| 25% | 1 | 3 | 3 | 30% | 1 | 3 | 30% |
| 50% | 2 | 2 | 2 | 20% | 2 | 4 | 40% |
| 75% | 3 | 1 | 1 | 10% | 3 | 3 | 30% |
| 100% | 4 | 0 | 0 | 0% | 4 | 0 | 0% |
| | | WAYS | 10 | 100% | WAYS | 10 | 100% |

| × 8/25 Ø | = | |
|---|---|---|
| 4 | 0 | 0% |
| 3 | 9 | 45% |
| 2 | 8 | 40% |
| 1 | 3 | 30% |
| 0 | 0 | 0% |
| WAYS | 20 | |

UPDATING



① Pose

LAUNCH CUSTOMER SERVICE ~ $D_1$ → $S_1$ LOW, $S_2$ HIGH } Risk of Predation

OR

DON'T LAUNCH ~ $D_2$ → $S_1$, $S_2$

Why? Customer Predation!

Launch New Service

2/0 Stand pat ~ BAU

J. Pearl Book why

0.013 ★ 0.987

| | | $Pr(S_1 \mid y)$ H0 | $Pr(S_2 \mid y)$ H1 |
|---|---|---|---|
| | | $S_1$ | $S_2$ |
| Launch | Do $D_1$ | 0 | $L(D_1 \mid S_2)$ |
| Don't launch | Do $D_2$ | $L(D_2 \mid S_1)$ | 0 ← Very small? |

→ $E(L \mid doD)$

② Form hypotheses

③ Sample market-tests

④ Count ⑤ Infer

⑥ Decide

4

LAUNCH or NOT? CAUSES

Beliefs about h criterion → pr(h)

| | $S_1$ | $S_2$ |
|---|---|---|
| $D_1$ | L | L |
| $D_2$ | L | L |

Samples h, costs ← data y from market samples

p-p model → $S_1$; $h_1$, $S_2$; $h_2$

$E(L \mid DoD_1) = pr(S_1 \mid y) L(S_1 \mid doD_1)$ → 0

$+ pr(S_2 \mid y) L(S_2 \mid doD_1)$

$= pr(S_2 \mid y) L(S_2 \mid doD_1)$

SO also

$E(L \mid doD_2) = pr(S_2 \mid y) L(D_2 \mid$

Do-$D_1$ means

actually LAUNCH Service

→ INVEST, PROMISE, ENGAGE, wait for the effects

WOLFCO → MOOSECO LAUNCH

Launch STAY or ? BAIL MOVE

$$E(L \mid do D_1) \overset{?}{<} E(L \mid do D_2)$$

$$pr(S_2 \mid y) L(S_2 \mid do D_1) \overset{?}{<} pr(S_1 \mid y) L(S_1 \mid do D_2)$$

$L(S_2 \mid do D_1) = 4$

$L(S_1 \mid do D_2) = 2$

**ODDS**

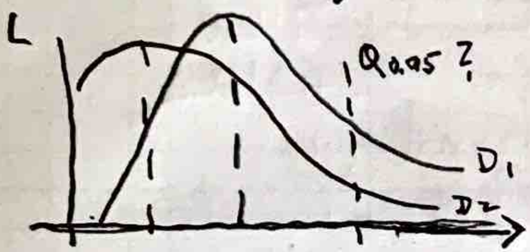
$$\frac{pr(S_1 \mid y) L(S_1 \mid do D_2)}{pr(S_2 \mid y) L(S_2 \mid do D_1)} > 1$$

$$\frac{(0.013)(2)}{(0.987)(4)} << 1$$

0.03 / 3.94

**CRITERION?**

e.g., Minimize Expected Loss (means someone might die?)




[Quantile are tail expectations!]

Objectively true value?

**Data**

Ex: $y \sim Normal(\mu, \sigma)$

$\mu = 2.5 <$ 1, 3

$\sigma = 1.0$

n = 3 samples

with $S_1 = 1$, $S_2 = 3$

use $pr(d \mid h)$

$$= \frac{1}{2\pi\sigma} \exp\left[-\left(\frac{y - \mu_h}{\sigma}\right)^2\right]$$

"h"

pr(s|y)

$Pr(3.87 \mid \mu = 1) = 0.006$ ✓

$pr(d_1 \mid h_1) P(h_1)$

(,000) d = y

| | h: μ = 1, 2 | p(h) | 3.87 | 1.94 | 2.86 | pr(d\|h) | pr(d\|h) pr(h) | pr(h\|d) |
|---|---|---|---|---|---|---|---|---|
| L0 | S1: μ = 1 | 0.5 | 0.006 | × 0.258 | × 0.070 | 0.00013 | 0.000015 | 0.013 |
| H1 | S2: μ = 3 | 0.5 | 0.273 | × 0.226 | × 0.395 | 0.02020 | 0.01010 | 0.987 |

pr(d|h)

0.01025

CONTINGENCY TABLES

"∧" = WEDGE → "AND" ★ ×
"∨" = "EITHER - OR" +

RATIONAL
PRINCIPLED

| | | Ø | 1 | |
|---|---|---|---|---|
| | | $d_1$ | $d_2$ | |
| NO | $h_1$ | 5 ($d_1 \wedge h_1$, $h_1 \wedge d_1$) | 3 | 8 |
| YES | $h_2$ | 2 | 7 | 9 |
| | | 7 | 10 | 17 |

$Pr(h_1) = \frac{8}{17}$, $Pr(d_1 | h_1) = \frac{5}{8}$

$Pr(h_2) = \frac{9}{17}$, $Pr(d_1 | h_2) = \frac{2}{9}$

$Pr(d_1) = \frac{7}{17}$ $Pr(d_2) = \frac{10}{17}$

$P(h_1 | d_1) = \frac{5}{7}$ $Pr(h_1 | d_2) = \frac{3}{10}$

COLUMN ↓ ROWS ↓

$Pr(h_1 \wedge d_1) = \frac{5}{17} = Pr(d_1 \wedge h_1)$

$Pr(d_1 | h_1) \, Pr(h_1)$

(∧)

$Pr(h_1 | d_1) \, Pr(d_1) = \frac{5}{17} = \frac{5}{8} \times \frac{8}{17}$

$\frac{5}{7} \times \frac{7}{17}$

★ = BAYES RULE

$$\frac{Pr(h_1 | d_1) \, Pr(d_1)}{Pr(d_1)} = \frac{Pr(d_1 | h_1) \, Pr(h_1)}{Pr(d_1)}$$

$$Pr(h_1 | d_1) = \frac{Pr(d_1 | h_1) \, P(h_1)}{Pr(d_1)} \quad (★)$$

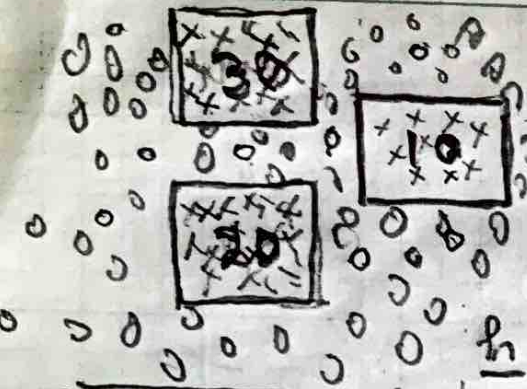
Infer

EX

$Pr(h_2 | d_1)$

$Pr(h_1 | d_2)$

$Pr(h_2 | d_2)$

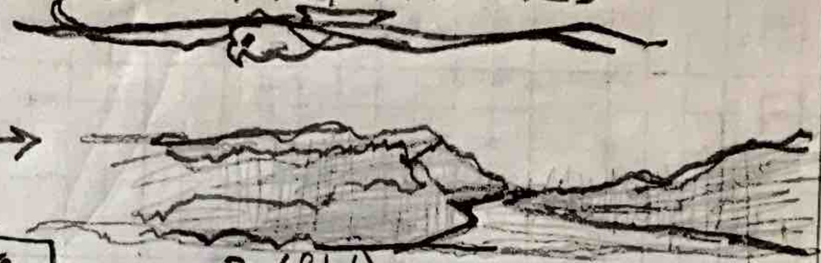



X's - O's

| n = 140 |
|---|
| $x_1 = 30$ |
| $x_2 = 10$ |
| $x_3 = 20$ |

d — SAMPLING AREA

# COUNTING RAPTORS



— BOTH X AND →

— EITHER / OR →

| h | λ | Pr(λ) | $x_1$ = 30 | $x_2$ = 10 | $x_3$ = 20 | | Pr(h\|d) |
|---|---|---|---|---|---|---|---|
| $\lambda_1$ | 0 | 0.2 | 0 | 0 | 0 | 0 | 0 |
| $\lambda_2$ | 10 | 0.2 | 0.0001 | 0.1251 | 0.0019 | 0.0...0 | 0.0...92 |
| $\lambda_3$ | 20 | 0.2 | 0.0083 | 0.0058 | 0.0888 | 0.0...36 | 0.9966 |
| $\lambda_4$ | 30 | 0.2 | 0.0726 | 0.0000 | 0.0134 | 0.0...00 | 0.0034 |
| $\lambda_5$ | 40 | 0.2 | 0.0135 | 0.0000 | 0.0002 | 0.0...0 | 0.0...0 |
| | | | | | | 0.0...87 | |

HIGHLY CONCENTRATED!

λ-GRID

| | |
|---|---|
| MAX | 40 |
| MIN | 0 |
| Y | 5 |
| E | (40−0)/(5−1) = 10 |

λ INTENSITY OF SIGHTINGS

$$= Pr(x|\lambda)$$

$$p = \frac{\lambda}{n} \Rightarrow \binom{n}{x}\left(\frac{\lambda}{n}\right)^x\left(1-\frac{\lambda}{n}\right)^{n-x} \Rightarrow e^{-\lambda}\frac{\lambda^x}{x!} = Pr(x|\lambda)$$

acts like k

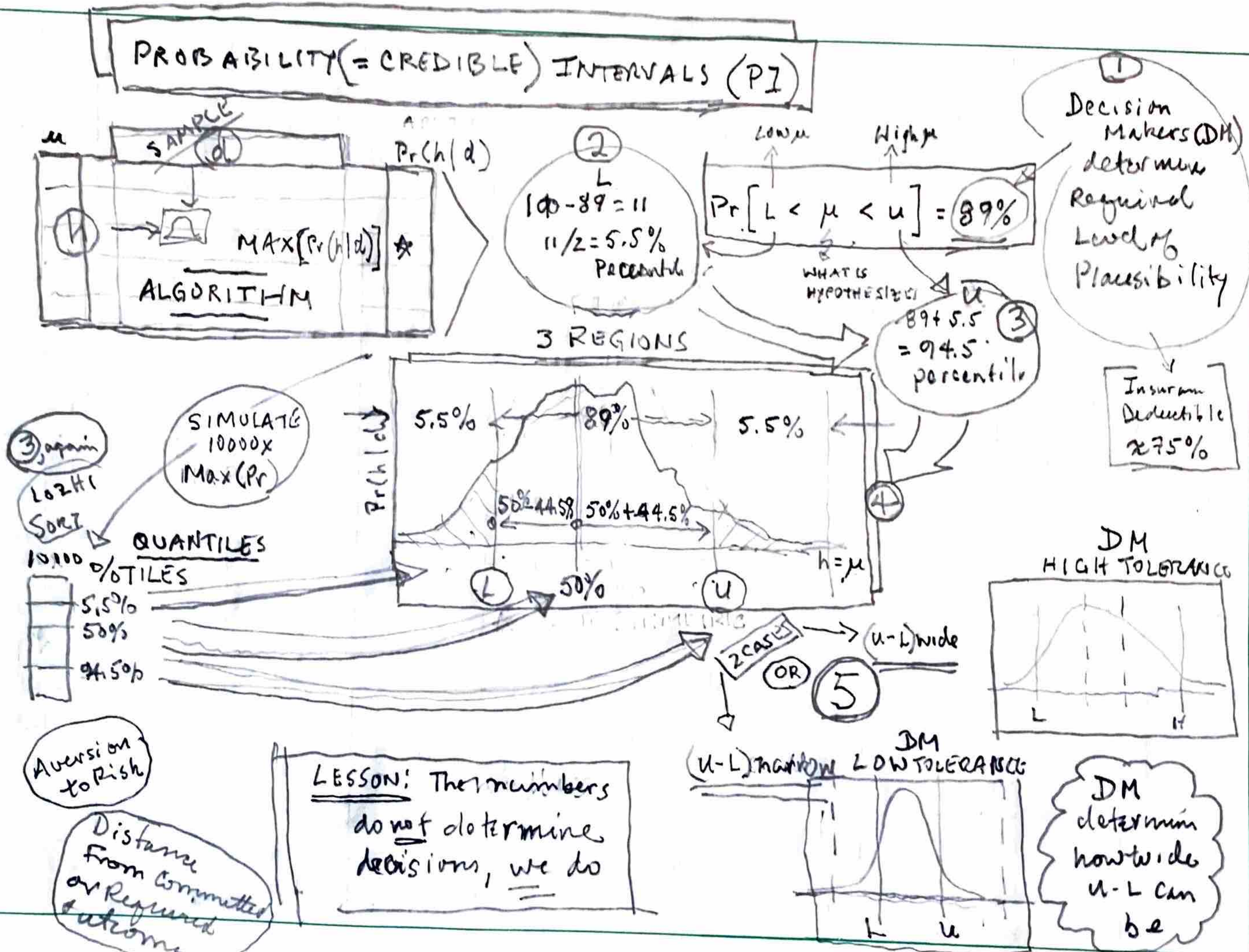
$$\frac{10}{140} \rightarrow \frac{1}{14} \Rightarrow \lambda = np = (140)(\ )$$

$$Pr(x=10|\lambda=\lambda_2=10) = e^{-10}\left(\frac{10^{10}}{10!}\right) = \ldots$$

$$(4.53999 \times 10^{-5})\left(\frac{1.0 \times 10^{10}}{3{,}628{,}800}\right) = 0.12511$$

by Calculator

POISSON


PROBABILITY (= CREDIBLE) INTERVALS (PI)
SAMPLE (d)
μ
MAX[Pr(h|d)]
ALGORITHM
Pr(h|d)
2
L
100-89=11
11/2=5.5%
Percentile
Low μ
High μ
Pr[L < μ < u] = 89%
WHAT IS HYPOTHESIZED
1
Decision Makers (DM) determine Required Level of Plausibility
u
89+5.5
=94.5
percentile
3
3 REGIONS
5.5%
89%
5.5%
50%-44.5%
50%+44.5%
Pr(h|d)
h=μ
L
50%
U
4
Insurance Deductible x75%
SIMULATE 10000x Max(Pr)
3, again
Lo2HI SORT
QUANTILES
10,000 %TILES
5.5%
50%
94.50%
2 CASES
(u-L) wide
OR
5
DM HIGH TOLERANCE
L
H
(u-L) narrow
DM LOW TOLERANCE
L
u
Aversion to Risk
Distance from Committee or Required outcome
LESSON: The numbers do not determine decisions, we do
DM determine how wide u-L can be

Assumptions → data, → model

| P | Pr(h\|d) |
|---|---|
| 0.00 | 0.000 |
| 0.25 | 0.065 |
| 0.50 | 0.348 |
| 0.75 | 0.587 |
| 1.00 | 0.000 |

Simple

Mutually Exclusive

Completely Exhaustive

Conjecture

MECE

From data → Pr(h=s|d); Normalize → Pr(s|d)

$0.065/0.4130$

Expected Contribution of $S_2$ to total Exp cost: $0.1579 \times 1200$

| States s | h=s Pr(h=s\|d) | → Pr(s\|d) | COST $D_1$=Stay | Expected Cost | $D_2$=Go | Expected Cost |
|---|---|---|---|---|---|---|
| Tolerable | 0.000 | 0.000 | 0 | 0 | 1700 | 0 |
| Uncomfortable | 0.065 | 0.1579 | 1200 | 189 | 1700 | 268 |
| Unacceptable | 0.348 | 0.8421 | 2100 | 1768 | 1700 | 1432 |
| | 0.4130 | 1.0000 | | 1958 | | 1700 |

Total Expected Cost

Min E(COST) $\{D_1, D_2\}$

subject to

→ Costs across states

→ States and data

→ Binomial model

"SHOULD I STAY OR SHOULD I GO?"

**GO**

IF YOU BELIEVE THIS ANALYSIS + DECISION ACHIEVES YOUR PURPOSE > BETTER THAN ALTERNATIVE

GREATER GOOD >

WAYS & MEANS

STAY OR GO

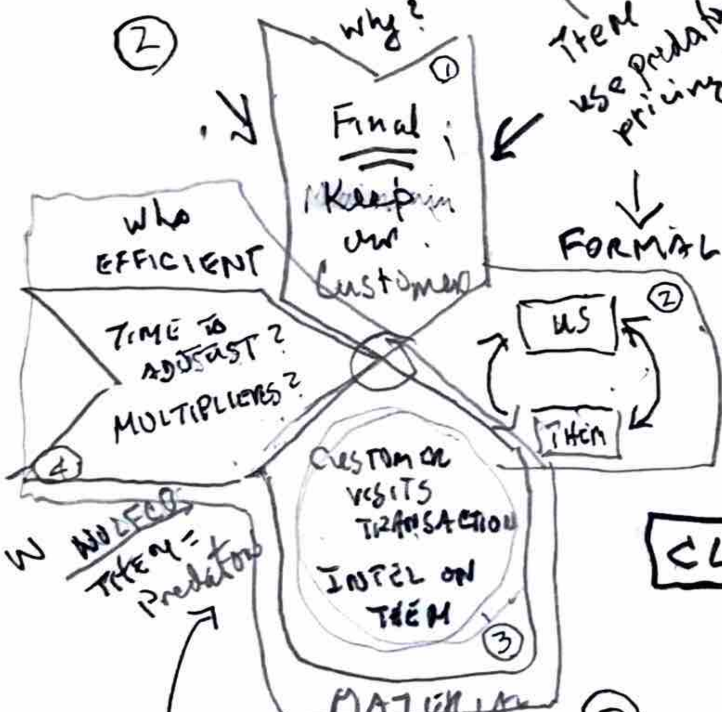
# WEEK 1

① **What if**

our customers can choose between Us or Them?

why?

② 

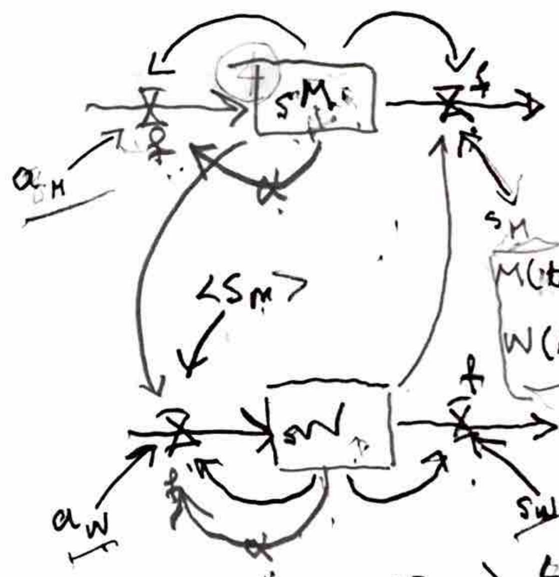



W WOLFCO THEM = Predator?

M MOOSECO US = prey

customer acquisition / customer switch

④




$t$: $\Delta t = t_s - t_r$

$t_r$ $t_s$

$t$ $t+\Delta t$

M or W: $\Delta M = M(t+\Delta t) - M(t)$

Let $\Delta t = 1$

$M(t+1) = M(t) + a_M M(t) - s_M M(t) W(t)$

$W(t+1) = W(t) + s_M M(t) a_W W(t) - s_W W(t)$

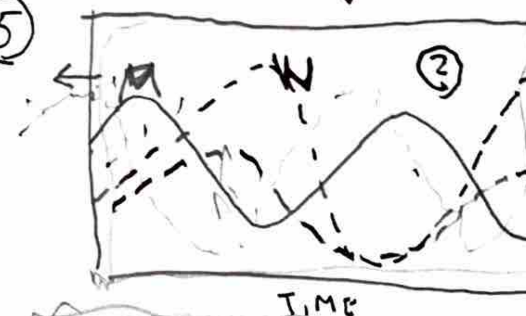
CLEANMETRICS.COM

R

ANALOGICALLY → **HBCo** Lynx/Hare / pelts

SYSTEMDYNAMICS101.COM

⑤

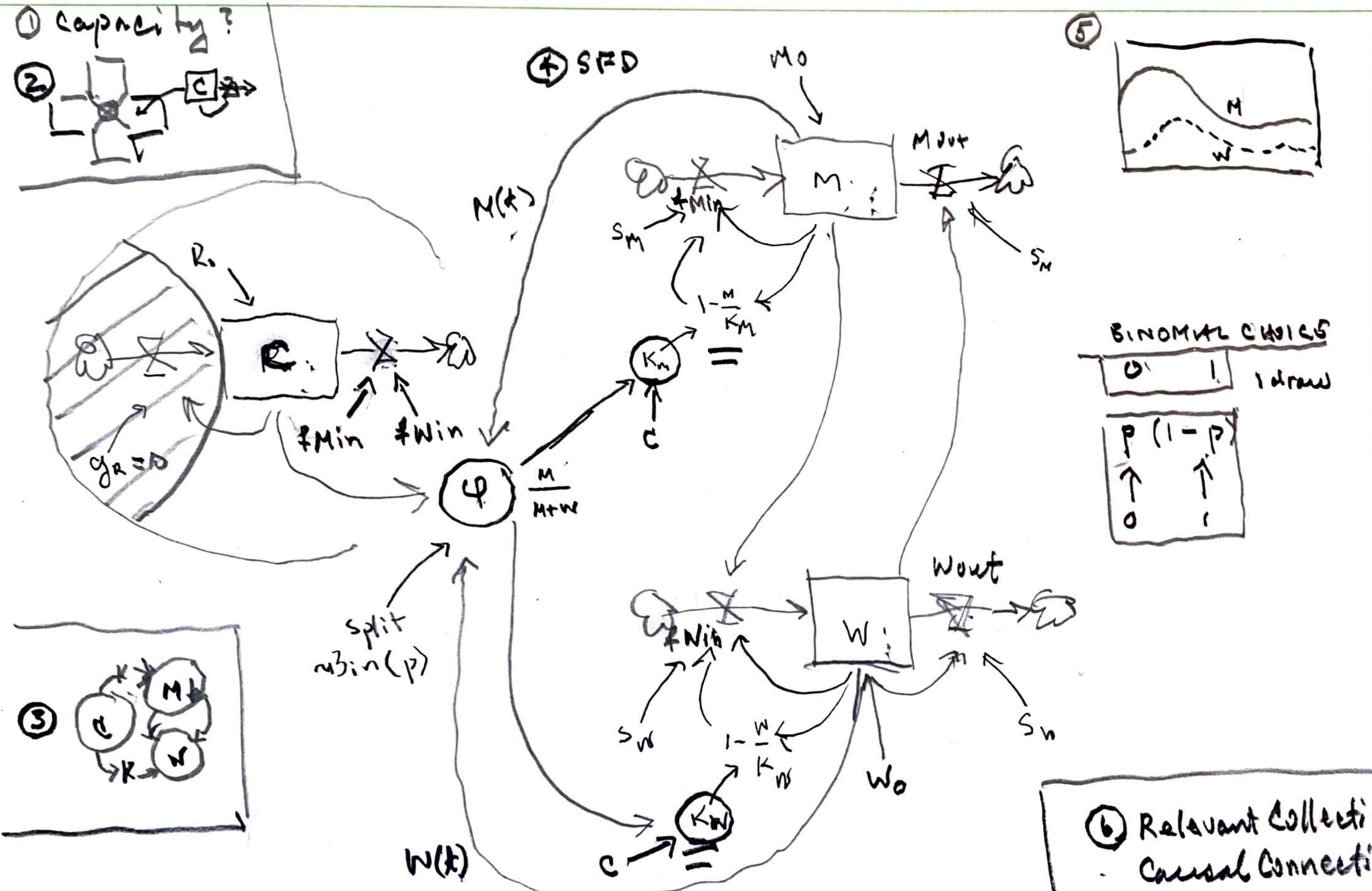



**CLD**


⑥ System?

- → Collection?
- → Connected?
- → Coherent?

LIMITS TO INTERACTIVE GROWTH




⑥ Relevant Collection

Causal Connection

Coherent Outcomes